Name:
Section:

MATH2216 Homework 1

Due in class Friday 9/7/18

(1) Please write the negations of the following statements, referring to Section 1.3 of the Hutchings notes if you need help. (BTW, no fair just writing "It is not the case that..." at the beginning of each statement. *Really* write down the *negation*, i.e., the statement that is true precisely when the original statement is false.) You needn't use symbols. English sentences are fine, as long as they mean what you mean them to mean.

(a) "All integers are even."

(b) "There exists an integer that is even."

(c) "There exists an $x \in \mathbb{Z}$ such that there exists a $y \in \mathbb{Z}$ with $x = 2y$."

(d) "For all $x \in \mathbb{Z}$, if there exists $y \in \mathbb{Z}$ such that $x = 2y + 1$ then there exists $z \in \mathbb{Z}$ such that $x^2 = 2z + 1$."

Solution:

(a)

(b)

(c)

(d)

(2) In the question above, notice that in each case either the *statement* was true or *the negation of the statement* was true. Indeed, this would have been the case no matter what statements I had chosen. Why is that? (Hint: What's the definition of a statement?)

Solution:

(3) Assume $k \in \mathbb{Z}^+$. Using the fact that $\sqrt{2} \notin \mathbb{Q}$, prove that $\sqrt{2^k} \in \mathbb{Q}$ iff ("if and only if", "$\Longleftrightarrow$") k is even. Recall that to prove an if and only if statement, you must prove both implications. In other words, you must prove:

(a) "only if", "$\Longrightarrow$": If $\sqrt{2^k} \in \mathbb{Q}$, then k is even.

(b) "if", "$\Longleftarrow$": If k is even, then $\sqrt{2^k} \in \mathbb{Q}$.

Solution:

(a)

(b)

(4) Assume $x \in \mathbb{Z}$. Prove that x is even iff x^3 is even.

Solution:

(5) (a) Let $x, y \in \mathbb{R}$. Prove that if $x \in \mathbb{Q}$ and $y \notin \mathbb{Q}$, then $x + y \notin \mathbb{Q}$.

(b) Let $x, y \in \mathbb{R}$. Does $x, y \notin \mathbb{Q}$ imply that $x + y \notin \mathbb{Q}$? If so, prove it. If not, find a counterexample.

(c) Let $x, y \in \mathbb{R}$. Does $x, y \in \mathbb{Q}$ imply that $x + y \in \mathbb{Q}$? If so, prove it. If not, find a counterexample.

Solution:

(a)

(b)

(c)

(6) Prove that

$$\sum_{k=1}^{2n} (-1)^k k = n$$

for all $n \in \mathbb{Z}^+$.

Solution: