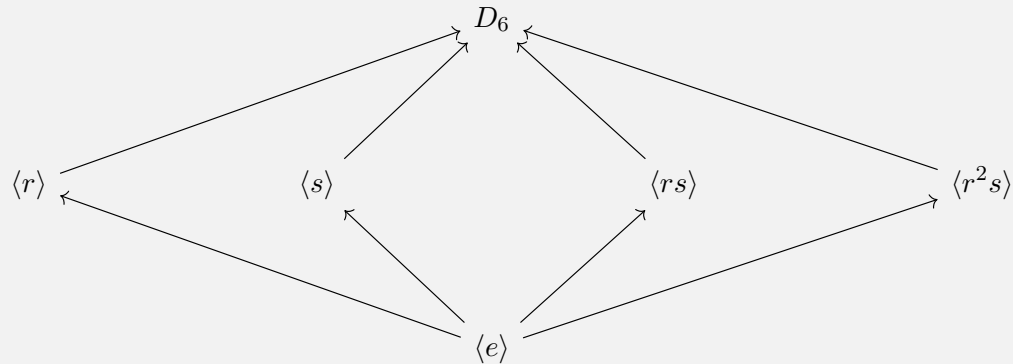


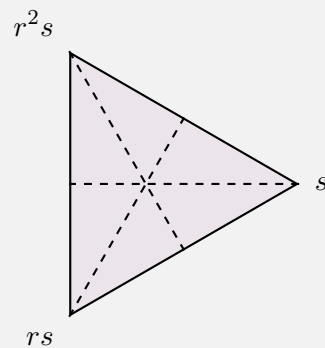
Lattice of subgroups for D_{12}

For dihedral groups, one can understand a lot about the meaning of subgroups by thinking about how they correspond to subshapes.

Example. $G = D_6$, the dihedral group of order 6.

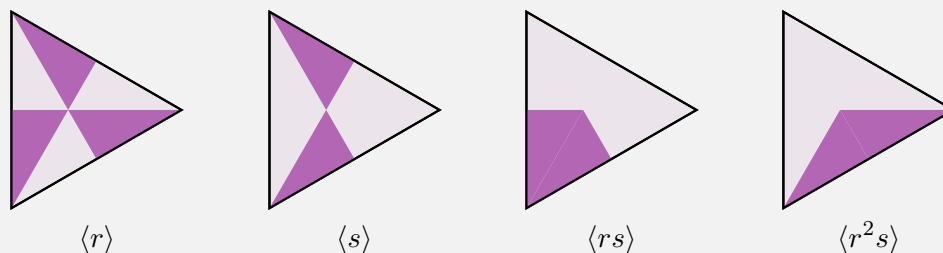


These are the reflection lines for the isometries s, rs, r^2s . To confirm that these are correct, use the fact the line of reflection is the unique line in the plane where the reflection restricts to the identity map. Check that you believe rs and r^2s restrict to the identity maps on the claimed lines!



(r rotates anti-clockwise...)

For each of the subgroups H in the lattice, we can find a subset Y of the triangle, whose isometry group $\text{Isom}(Y)$ is isomorphic to the subgroup H . There are lots of ways to do this! For example:



Exercises. Here are some exercises to repeat the D_6 example for more interesting group

$$D_{12} = \{e, r, r^2, r^3, r^4, r^5, s, rs, r^2s, r^3s, r^4s, r^5s\}.$$

1. Here are the subgroups of D_{12} . Fill in arrows to complete the subgroup lattice.

$$D_{12}$$

$$\langle r^2, s \rangle$$

$$\langle r \rangle$$

$$\langle r^2, rs \rangle$$

$$\langle r^3, s \rangle$$

$$\langle r^3, r^2s \rangle$$

$$\langle r^3, rs \rangle$$

$$\langle r^2 \rangle$$

$$\langle s \rangle$$

$$\langle r^2s \rangle$$

$$\langle r^4s \rangle$$

$$\langle r^3 \rangle$$

$$\langle rs \rangle$$

$$\langle r^3s \rangle$$

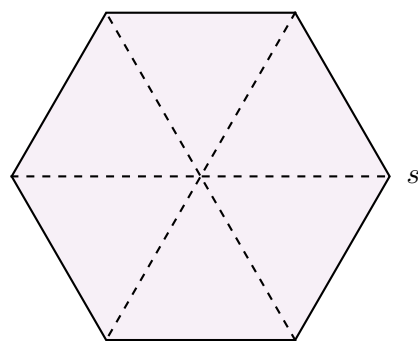
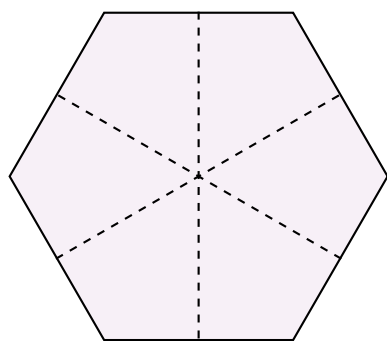
$$\langle r^5s \rangle$$

$$\langle e \rangle$$

2. Label the reflection lines for the hexagon with the five remaining reflections

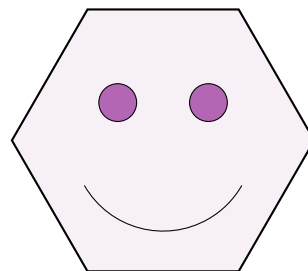
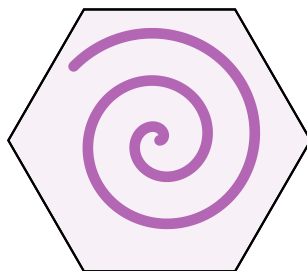
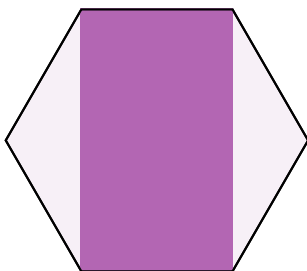
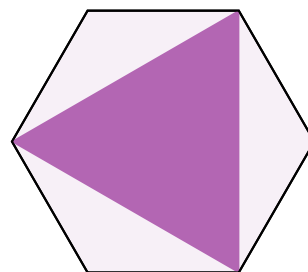
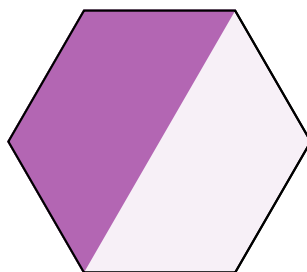
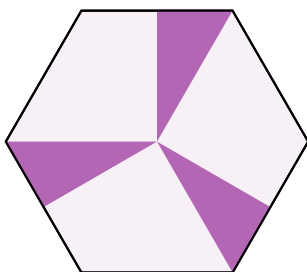
$$rs, r^2s, r^3s, r^4s, r^5s.$$

(Each of the reflections will fix exactly one line – use this to deduce the labelling.)



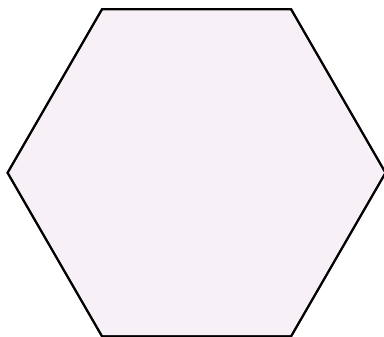
(r rotates anti-clockwise...)

3. Here are some subsets Y of the hexagon. In each case, decide which subgroup H of D_{12} is isomorphic to $\text{Isom}(Y)$.

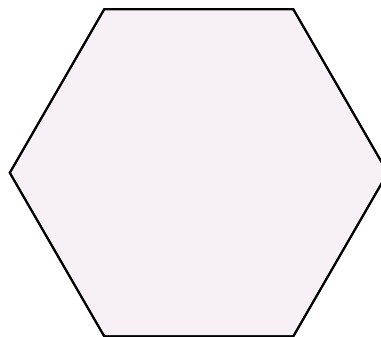


4. For each given subgroup $H \leq D_{12}$, shade in your own subset Y whose isometry group is isomorphic to that H .

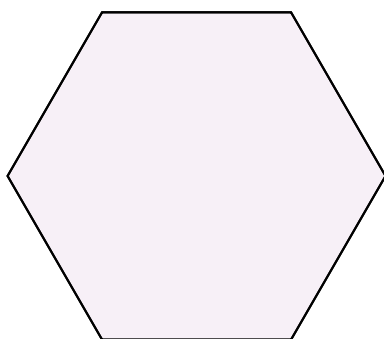
- Your subset Y should be preserved by every element of the given subgroup $H \leq D_{12}$.
- There should be no $h \in D_{12} \setminus H$ that preserves your subset Y ; in other words, your subset should not possess any extra dihedral symmetries.



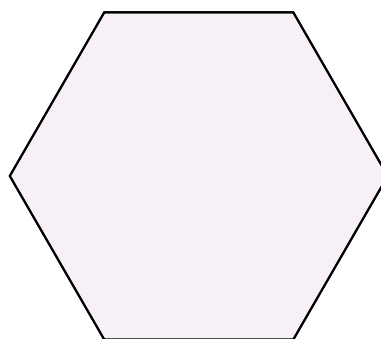
$$H = \langle r^3 \rangle = \{e, r^3\}$$



$$H = \langle r^3, rs \rangle = \{e, r^3, rs, r^4s\}$$



$$H = \langle r^2, rs \rangle = \{e, r^2, r^4, rs, r^3s, r^5s\}$$



$$H = \langle e \rangle = \{e\}$$